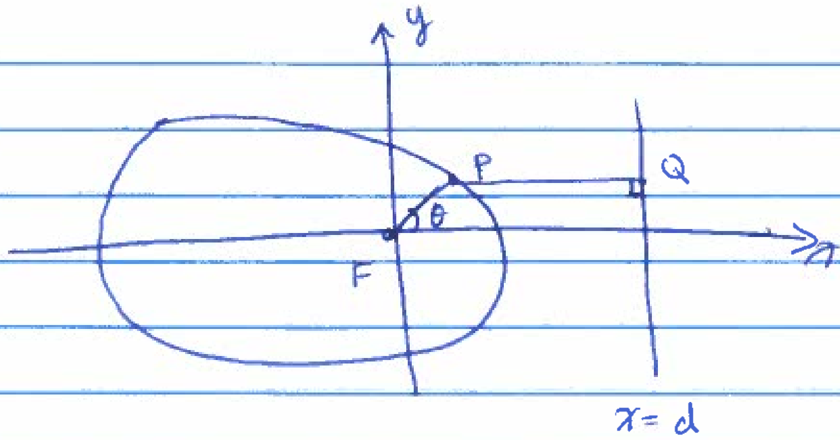
Apr. 17, 2024



By definition, an ellipse is a set of points P where:

$|FP| = e\,|PQ|$ where e is the eccentricity, and $0 < e < 1$

F is the focus and d is the directrix.

Let F be at the origin, then:

$r = e(d - r\cos\theta)$ $\quad (r = |FP|,\ d - r\cos\theta = |PQ|)$

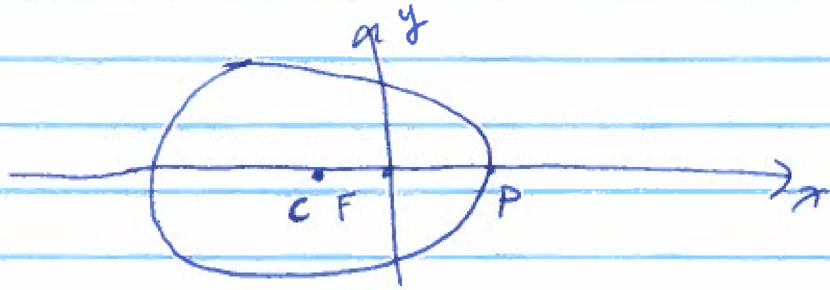
$r = ed - re\cos\theta$

$r + re\cos\theta = ed$ $\quad \Rightarrow \quad \boxed{r = \dfrac{ed}{1 + e\cos\theta}}$

Polar equation of an ellipse with one focus at the origin.

To write the Polar equation, $r(\theta) = \dfrac{ed}{1 + e\cos\theta}$,

in terms of ellipse's semi-major axis a, consider the following:



Here the ellipse has the origin focus being on the "right" side of the ellipse, and so center of ellipse, C, is to the left of F. By definition, $|CF|$ is the linear eccentricity of the ellipse; $|CP| = a$; and $e = \dfrac{|CF|}{|CP|}$

Also, $|FP| = r(0^\circ) = \dfrac{ed}{1+e}$. Putting everything together

$$a = |CP| = |CF| + |FP|$$
$$= ae + \frac{ed}{1+e}$$

$$a(1-e) = \frac{ed}{1+e} \Rightarrow ed = a(1-e)(1+e)$$
$$= a(1-e^2)$$

$$\boxed{r(\theta) = \frac{a(1-e^2)}{1+e\cos\theta}}$$